Geometric Mean and Standard Deviation

The geometric mean and standard deviation should be used for investments that have a maturity of one year or more because it accounts for the compounding that occurs over multiple years. Assuming that you are using it for an investment with "P" number of periods per year and that spans "n" years, the following equations can be used to derive the annual geometric mean and standard deviations of the investment's returns. Note that if the number of periods is only 1 (i.e., you are using annual returns in your analysis), then the equations collapse to just using "n" the number of years:

Geometric Mean (GM)

$$GM = \left[\left(\prod_{i=1}^{n \times P} (1 + r_i) \right)^{\frac{1}{n \times P}} - 1 \right] P$$

Or,

$$GM = \left[\exp\left(\frac{1}{(n \times P)} \sum_{i=1}^{n \times P} \ln(1 + r_i) \right) - 1 \right] P$$

Or,

$$GM \equiv CAGR = \left(\left(\frac{V_{n \times P}}{V_i} \right)^{\frac{1}{n \times P}} - 1 \right) P$$

CAGR stands for compound annual growth rate, and it is equivalent to the geometric mean.

Geometric Standard Deviation (GSD)

$$GSD = \left[\exp\left(\frac{1}{(n \times P - 1)} \sum_{i=1}^{n \times P} \left(\ln(1 + r_i) - \ln\left(1 + \frac{GM}{P}\right) \right)^2 \right)^{\frac{1}{2}} - 1 \right] P^{\frac{1}{2}}$$

Note that sample statistics are being applied since the 1 (one) has been subtracted from all of the "P" and "n×P" denominators for the standard deviation calculations.

Definitions:

n = number of years
P = number of periods per year
r = periodic return
V = Value of financial asset (e.g., index or stock price)
ln() = natural log function
exp() = Exponential function